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Experimental Static Analysis of a Cantilever Beam with Nonlinear Parameters

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Abstract

The beam-like structures are typically subjected to dynamic loads. In this paper classical problem of deflection of a cantilever beam of linear elastic material, under the action of a uniformly distributed load along its length (its own weight), is experimentally and numerically analyzed. Paper presents the differential equation governing the behavior of this system and shows that these equations are difficult to solve due to the presence of nonlinear term. The experiment described in this paper is an easy way to introduce the concept of geometric nonlinearity in mechanics of material. Finally numerical result is carried out by ANSYS program and comparedwith the experimental results.

Keywords: Cantilever, Non-linear, Numerical analysis, Experimental analysis

Introduction

Vibration analysis is very important in the designing of structural and mechanical system. For beams undergoing small displacements, linear beam theory can be used to calculate the natural frequencies, mode shapes, and the response for a given excitation. However, when the displacements are large, linear beam theory fails to accurately describe the dynamic characteristics of the system. Highly flexible beams, typically found in aerospace applications, may experience large displacements. These large displacements cause geometric and other nonlinearities to be significant. The nonlinearities couple the modes of vibration and can lead to modal interactions where energy is transferred between modes [1]:

In reality, no physical system is strictly linear and hence linear models of physical systems have limitations of their own. In general, linear models are applicable only in a very restrictive domain like when the vibration amplitude is very small. Thus, to accurately identify and understand the dynamic behavior of a structural system under general loading conditions, it is essential that nonlinearities present in the system also be modeled and studied.

The objective of this study is to analyze the free and forced vibration behavior of beams by considering nonlinear parameters.

Theorotical analysis

Let us consider the case of a long, thin, cantilever beam of uniform rectangular cross section made of a linear elastic material, whose weight is *W*, subjected to a tip load *F* as shown in Fig. In this study, it is assumed that the beam is non-extensible and the strains remain small. Firstly, assume that Bernoulli-Euler's hypothesis is valid, i. e., plane cross-sections which are perpendicular to the neutral axis before deformation remain plane and perpendicular to the neutral axis after deformation. It is also assumed that the plane-sections do not change their shape or area.

The Bernoulli-Euler bending moment-curvature relationship for a uniformsection rectangular beam of linear elastic material can be written as follows:

$$M = EIk \qquad (1)$$

Where *E* is the Young's modulus of the material, *M* and *k* are the bending moment and the curvature at any point of the beam, respectively, and *I* is the moment of inertia (the second moment of area) of the beam cross-section about the neutral axis [2] and [3]. The product *EI*, which depends on the type of material and the geometrical characteristics of the cross-section of the beam, is known as the flexural rigidity.

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The moment of inertia of the cross section is given by the equation [4]:

$$I=1/12(bh3) \quad (2)$$

And its value for the cantilever beam by experimental analysis is I = 1.333 x 10-13m^4.

Equation (1) which involves the bending moment, M- governs the deflections of uniform rectangular cantilever beam made of linear type material under general loading conditions. Differentiating equation (1) once with respect to s, one can obtain the equation that governs large deflections of a uniform rectangular cantilever beam:

$$\frac{dk}{ds} = \frac{1}{EI}\frac{dM}{ds} \quad (3)$$

Let dx and dy are the horizontal and vertical displacements at the free end of the beam and φ_0 takes into account the maximum slope of the beam. We take the origin of the Cartesian coordinate system at the fixed end of the beam and let (x,y) be the coordinates of point A, and s the arc length of the beam between the fixed end and point A. The bending moment M at a point A with Cartesian coordinates (x,y) can be easily calculated from the equation

$$M(s) = \int_s^L w[x(u) - x(s)] + F(L - dx - x) \quad (4)$$

Where L – dx – x is the distance from the section of the beam at a point A to the freeend where force F is applied, and u is a dummy variable of s. Differentiating equation (4) once with respect to s and recognizing that cos φ = dx/ds [5]:

$$\frac{dM}{ds} = -w(L-s)cos\varphi - Fcos\varphi \quad (5)$$

Using equation (5), in equation (2) becomes:

$$\frac{d^2_{\varphi(s)}}{ds^2} = -\frac{1}{EI}[w(L-s) + F]\ cos\ \varphi(s) \quad (6)$$

By taking relation between k and φ **into account**

$$\mathbf{K = \frac{d\varphi}{ds}} \quad (7)$$

Equation (6) is the non-linear differential equation that governs the deflections of a cantilever beam made of a linear material under the action of a uniformly distributed load and a vertical concentrated load at the free end. The boundary conditions for equation (6) are:

$$\varphi(\mathbf{0}) = 0 \quad (8)$$

$$\left(\frac{d\varphi}{ds}\right)_{s=L} = 0 \quad (9)$$

Taking into account that cosφ = dx/ds and sinφ = dy/ds, the x and y coordinates of any point of the elastic curve of the cantilever beam are found asfollows

$$x(s) = \int_0^s cos\varphi(s)ds \quad (10)$$

$$y(s) = \int_0^s sin\varphi(s)ds \quad (11)$$

The horizontal and vertical displacements at the free end can be obtained from equations (10) and (11) for s = L [6]:

$$dx = L - x(L) \quad (12)$$

$$dy = L - y(L) \quad (13)$$

Experimental set up for load deflection curve

Fig.1 shows a photograph of a system made up of a steel beam of rectangular cross section fixed at one end and loaded at the other end. The beam is fixed to the C channel by means of a clamp, which provides a better support as shown in Fig.1.The experimental measurements of the elastic curve of the beam as well as the horizontal and vertical displacement at the free end of the beam are obtained on this set up.

Fig.1 Cantilever Beam Under the Action of a Uniformly Distributed Load and a Vertical Concentrated Load at the Free End

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The experimental parameters are shown in Table1.

Table 1 Experimental Parameters

Sr. No.	Parameter	Values
1	Length of beam	0.09 m
2	Width of beam	0.025 m
3	Height of beam	0.005 m
4	Weight of beam	8.83N

Numerical analysis

The large deflections of a cantilever beam are obtained by using ANSYS program; a comprehensive finite element package is used. The ANSYS/Structural package that simulates both the linear and nonlinear effects of structural models obtained a dynamic environment. Firstly Young's modulus of the material is obtained to do this experimental the values of the vertical displacements obtained at free end, $\mathbf{d_y}$, for different values of the concentrated load F applied at the free end of the beam and obtained. The seven values for F: 4.905, 9.81, 14.715, 19.62, 24.525 and 29.43 N and to obtain the theoretical value of $\mathbf{d_y}$ for different values of E around the value of E=200 GPa (the typical value of Young's Modulus for Mild Steel) using the ANSYS program. The Young's modulus E by comparing the experimentally measured displacements at the free end $\mathbf{d_{y,exp}}$, (Fj), where j = 1,2,…,J; J being the number of different external loads F considered (in our analysis J = 7), with the numerically calculated displacements $\mathbf{d_y}$ (E,Fj). We obtain the value of E for which the sum of the mean square root x^2 is minimum, where x^2 is given by the following equation [4]:

$$x^2(E) = \sum_{j=1}^{J}[d_y(E,F_j) - (F_j)]^2 \quad (14)$$

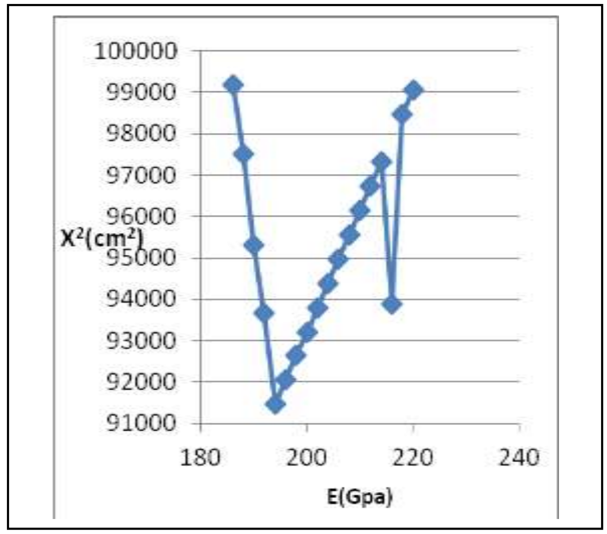
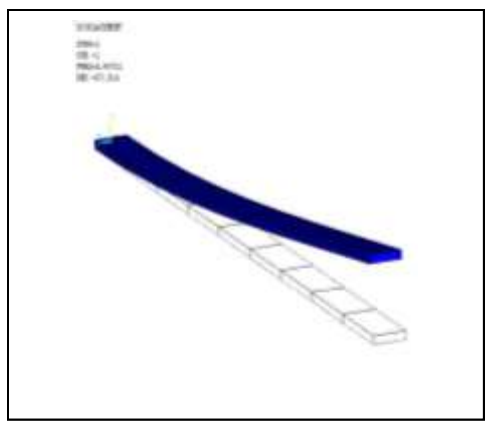


Fig.2.Calculated Values of x^2 as a Function of E.

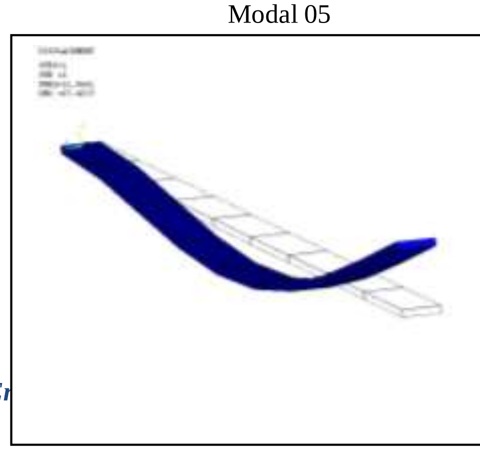
In Fig. 2 we have plotted the calculated values of x^2as a function of E. The value of Young's Modulus that minimizes the quantity x^2 is E=194 GPa, which implies that the flexural rigidity is EI =0.050 Nm2.

The numerical results obtained for nonlinear MS beam are as follows for E=194 GPa

Modal 01



Modal 05



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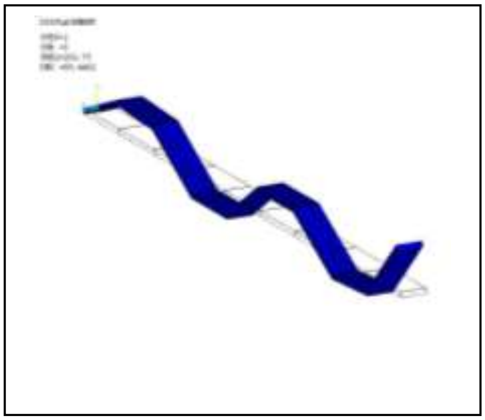
Modal 06



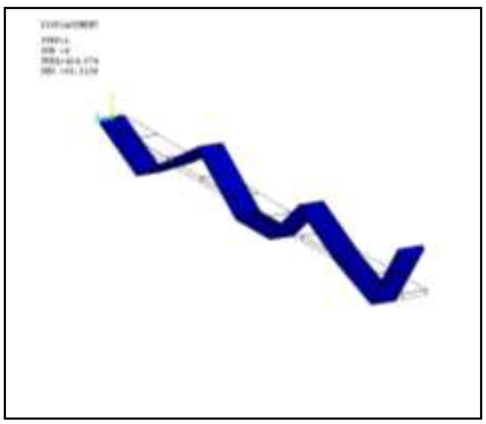


Fig 3.Modal Analysis Results obtained for Non Linear MS Material Beam

Experimental set up for static analysis

Fig.3 shows Experimental Set up for Static Analysis, in which proper connections of accelerometer, modal hammer, laptop and FFT Analyzer were made. Then by bump test various frequencies are obtained with the help of FFT Analyzer.

Result and Discussion

Table 2 shows the values of frequency in Hz calculated numerically with the aid of the ANSYS program. The relative error of the frequency is calculated by comparing these values with experimental results.

Table 2 Relative Error in Frequency obtained by Experimental and Numerical Analysis

Sr. No	Experimental Frequency in Hz (Static Test)	Numerical Frequency in Hz (for 194 GPa)	Relative Error in Percentage
1	5.033	4.9571	1.5%
2	31.5414	31.066	1.5%
3	88.3115	87.010	1.4%
4	173.041	170.67	1.3%
5	286.017	282.75	1.1%
6	427.03	424.07	0.93%

This shows that the relative error in percentage of the frequency between numerical and experimental analysis is about 1.2 %.

Conclusion

Comparative static analysis of cantilever beam for mild steel material is carried out. Nonlinearities present in cantilever beam are found out. The numerical results from Finite Element analysis showed in general a good agreement with the experimental static values. However, differences appear indicating the necessity to improve the model input data and the experimental procedure as well as due to nonlinearities present in the system.

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